A Constraint Hierarchies Approach to Geometric Constraints on Sketches

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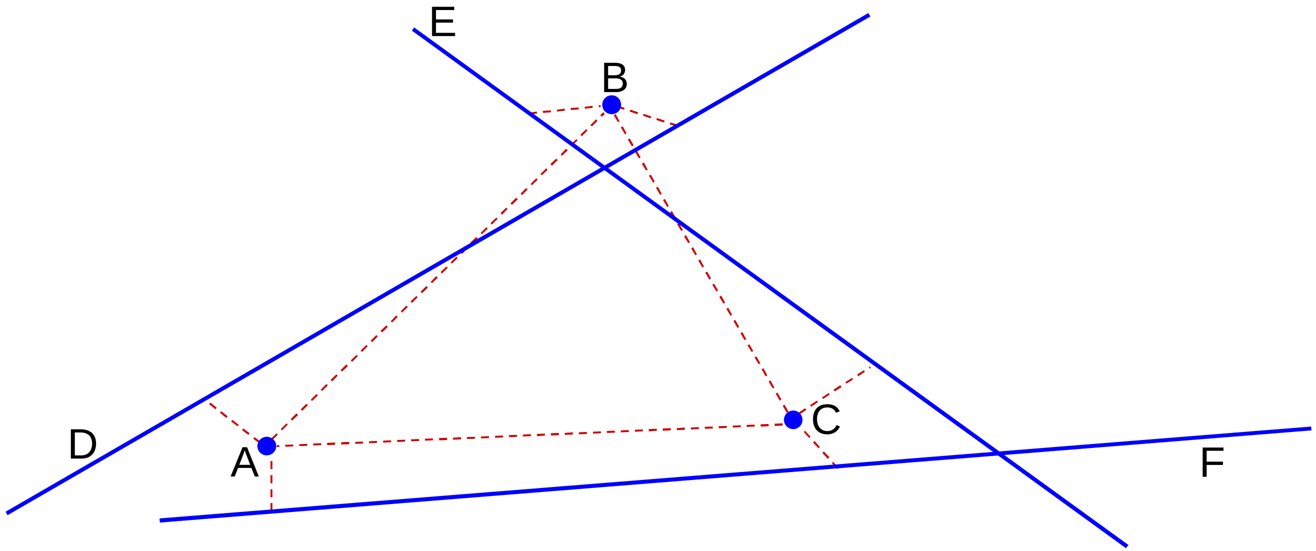
Motivation

- A geometric constraint system in 2D:
  - Entities: 3 points A,B,C, 3 lines D,E,F
  - Constraints: 3 pt-pt distances, 6 pt-ln incidences



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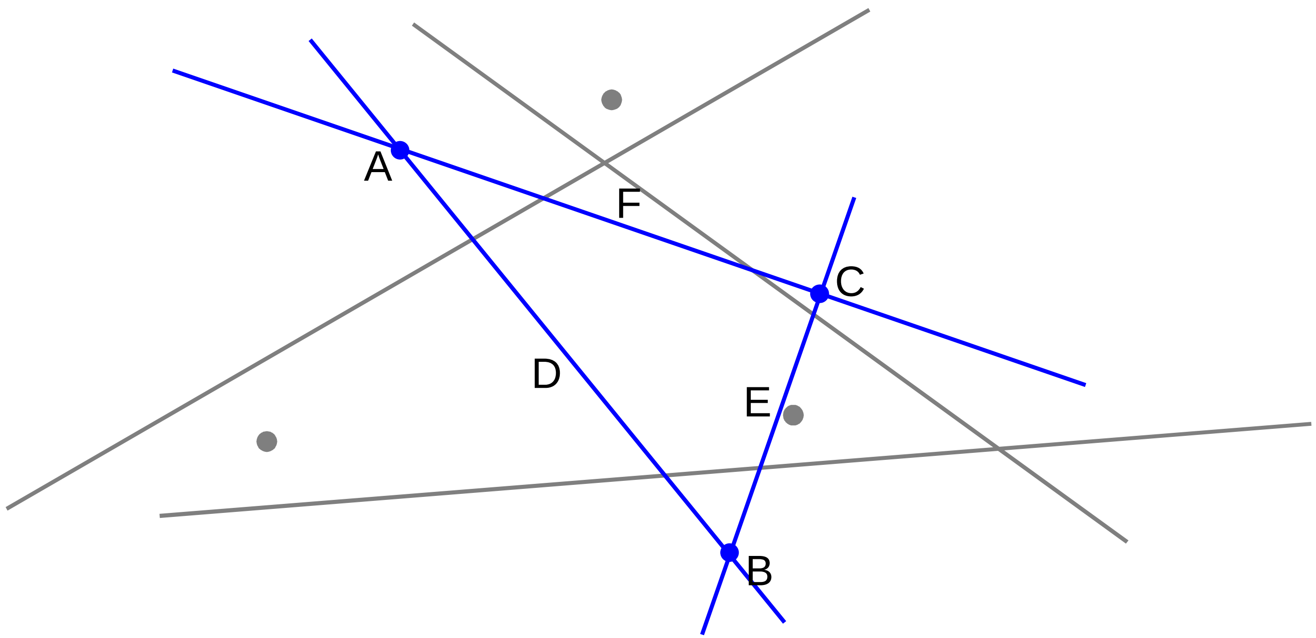

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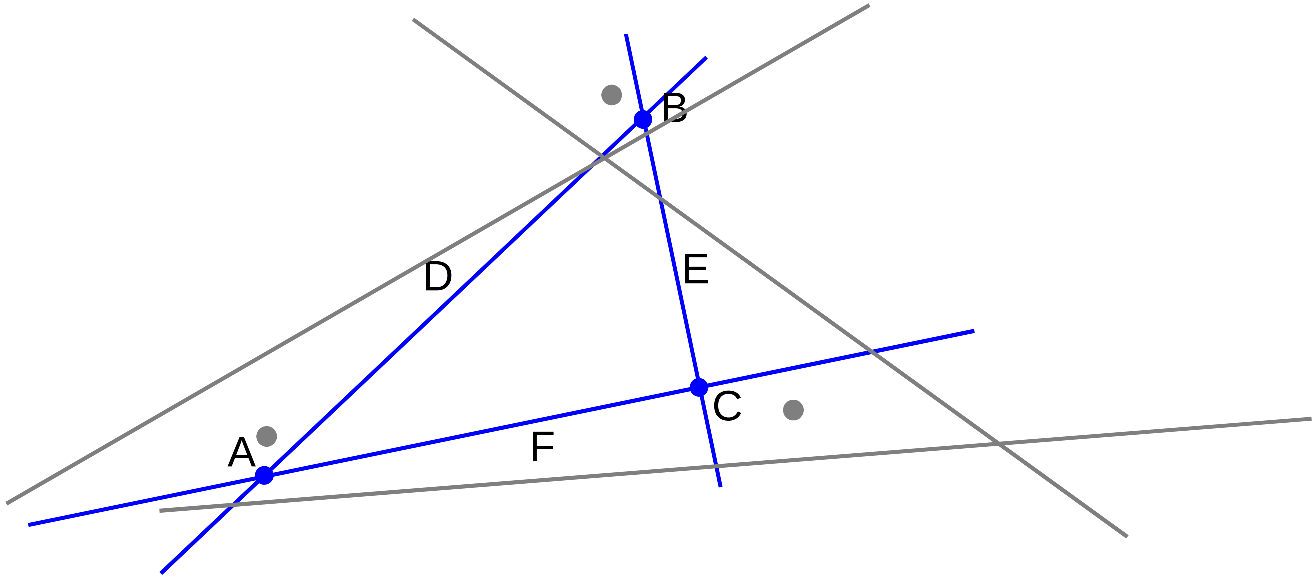

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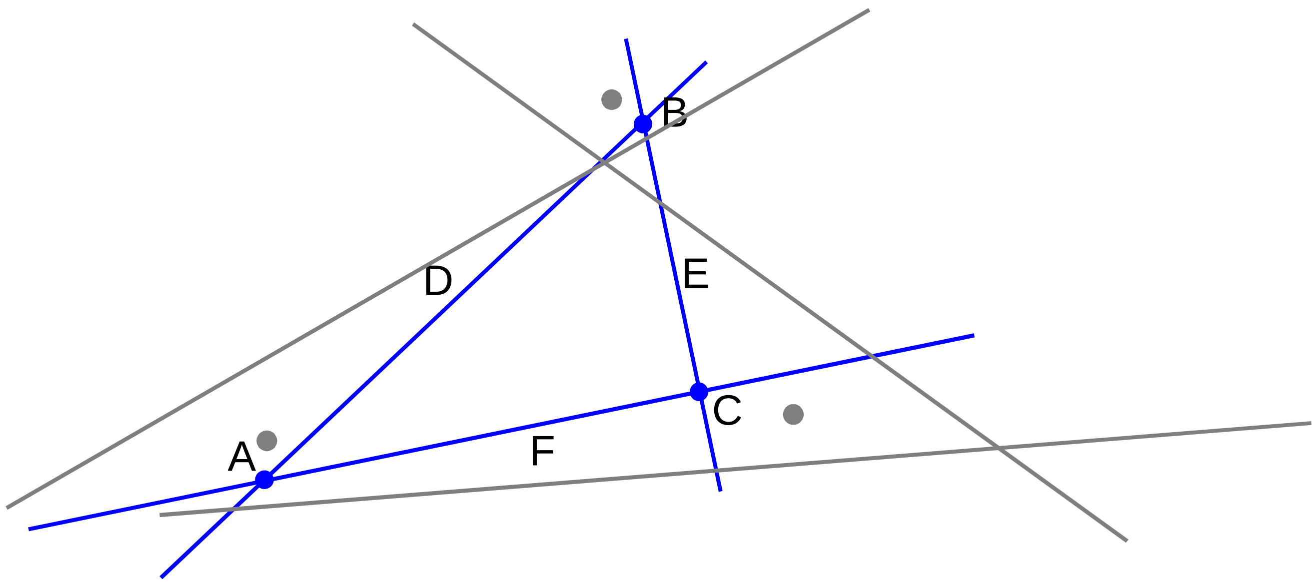
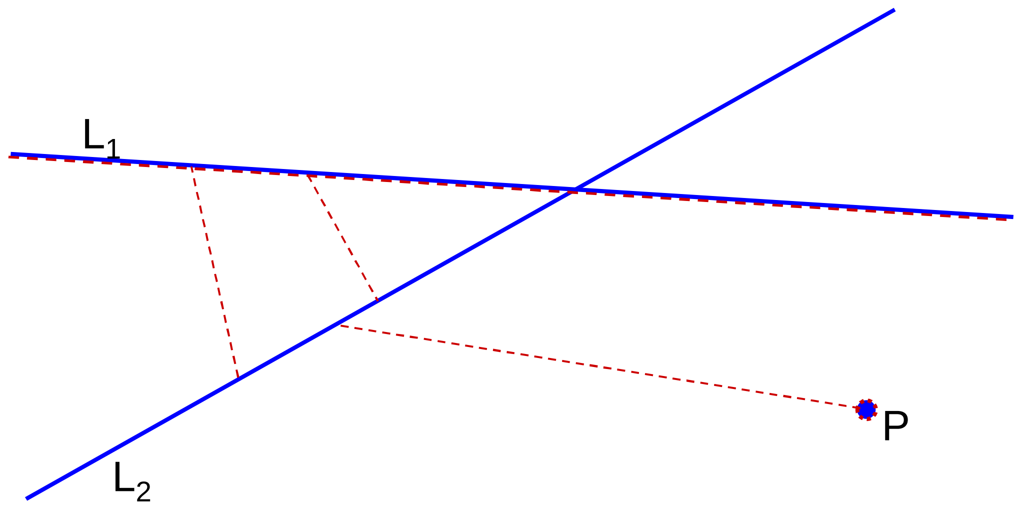


The sketch expresses the designer's intents

→ Solvers should take the sketch into account!

Motivation

- A geometric constraint system in 3D:
  - Entities: 1 point P, 2 lines L_1, L_2
  - Constraints: Fixed(P), Fixed(L_1), Parallelism(L_1,L_2), Incidence(P,L_2), Ortho_distance(L_1,L_2,4)



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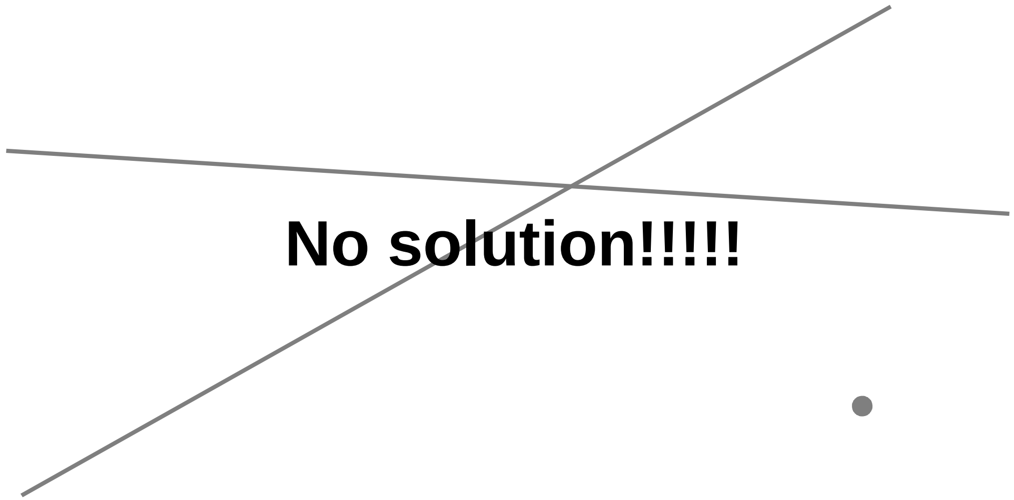

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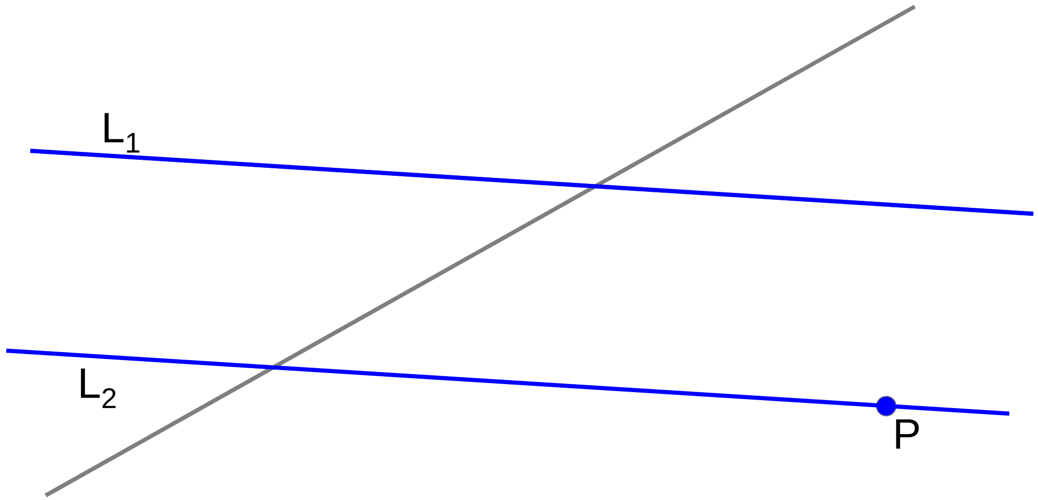

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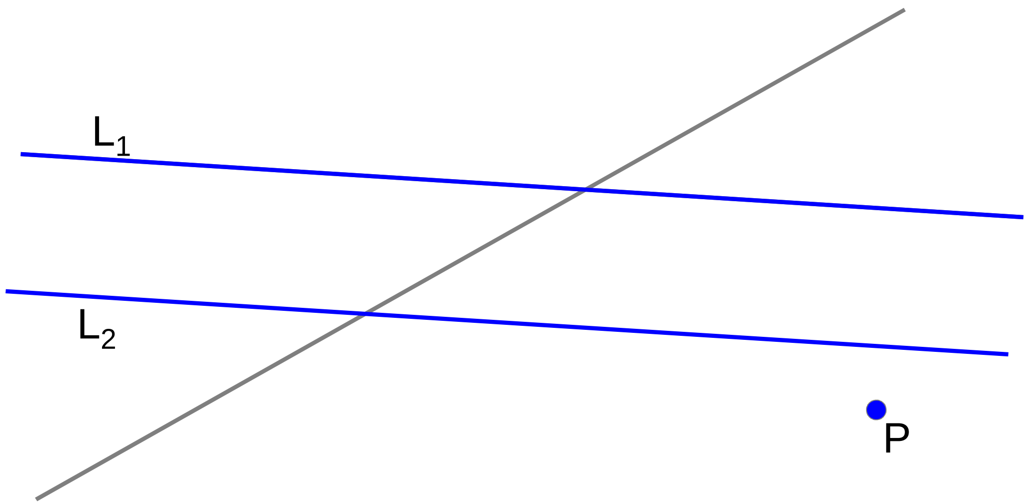

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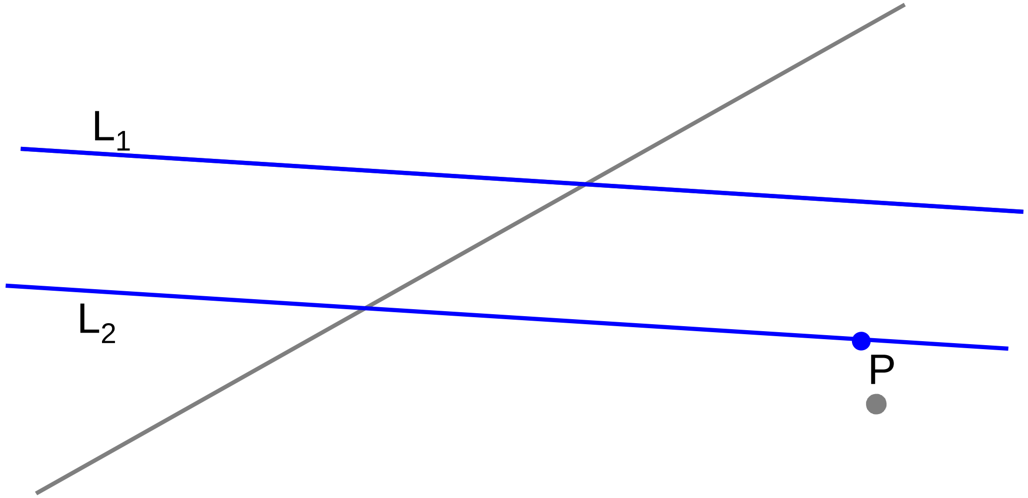

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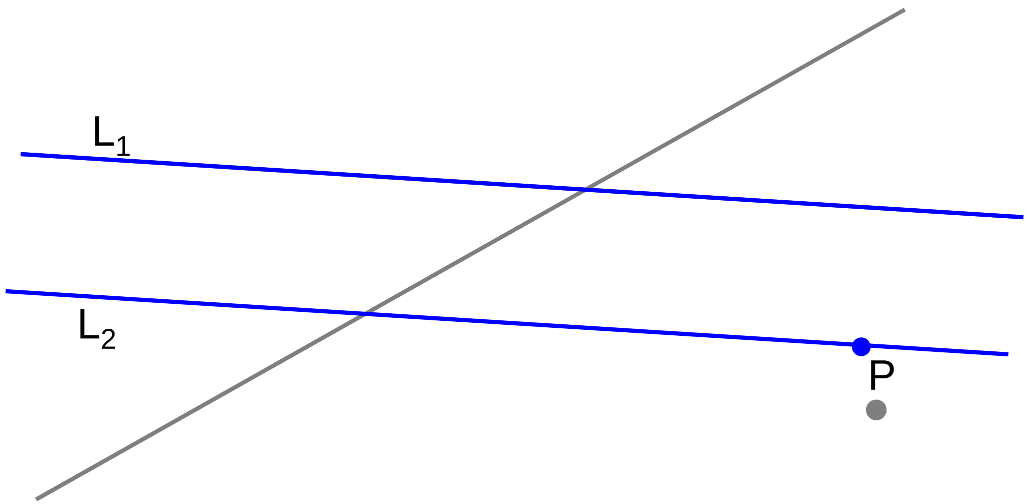


Debugging a geometric constraint system is tedious
→ Solvers should relax constraints as needed!

Outline

Background

1. Geometric Constraints Systems
   - Definitions
   - Considered Method
2. Constraint Hierarchies
   - Definitions
   - Considered Method

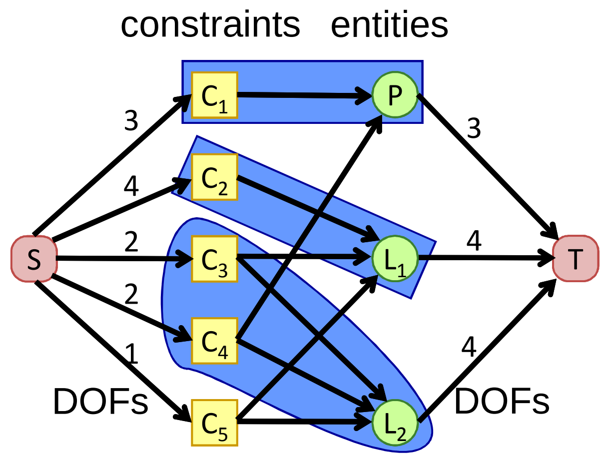
Contribution

3. Geometric Constraint Hierarchies
   - Definitions, advantages
   - Proposed Method
4. Conclusions
   - Discussion
   - Future work

1- Geometric Constraints Systems

- Geometric constraint systems (GCSs) = entities + constraints (w.r.t. a geometric universe)
- Solution = configuration of the entities that satisfies all the constraints
- Considered solving method: flow-based decomposition-recombination planner (+ numerical evaluation / IBB)
  [Hoffmann et al. 97-00, Jermann et al. 03]

  Example:
  - Entities:
    Point P, Lines L_1, L_2
  - Constraints:
    c1: Fixed(P)
    c2: Fixed(L_1)
    c3: Parallelism(L_1,L_2)
    c4: Incidence(P,L_2)
    c5: Ortho_distance(L_1,L_2,4)



Failure : no solution returned!

2- Constraint Hierarchies

- Constraint Hierarchies = variables + constraints + strengths + solution criterion
- Solution = an assignment that is not dominated by any other w.r.t. the solution criterion
  - locally-predicate-better (LPB) = satisfied constraints set inclusion in each hierarchy level considered in decreasing strength order

<u>Example:</u>

- Variables:

  $x_1, x_2, x_3 \in \{0,1,2,3\}$

- Constraints:

required	c_1: $x_1 = x_2$
strong	c_2: $x_2+1 = x_3$
weak	c_3: $x_1 = 0$
weak	c_4: $x_3 = 3$

- Assignments:

  $(x_1,x_2,x_3) \rightarrow$ *(sat. const.)*

  a_1: $(0,2,3) \rightarrow (\emptyset,\{c_2\},\{c_3,c_4\})$

  a_2: $(0,0,3) \rightarrow (\{c_1\},\emptyset,\{c_3,c_4\})$

  a_3: $(2,2,3) \rightarrow (\{c_1\},\{c_2\},\{c_4\})$

  a_4: $(0,0,1) \rightarrow (\{c_1\},\{c_2\},\{c_3\})$

- LPB-comparison:

  a_1 violates c_1

  $\rightarrow$ not a solution

  $a_2 <_{LPB} a_3$

  $\rightarrow$ not an LPB-sol.

  $a_3 \cancel{<_{LPB}} a_4$, $a_4 \cancel{<_{LPB}} a_3$

  $\rightarrow$ both LPB-sol.

2- Constraint Hierarchies

- Considered solving method: maximum matching based identification of a maximal set of satisfiable constraints per hierarchy level (LPB-maximal set of constraints) [Gangnet and Rosenberg, 93]

  Example:

  - Variables:

    $x_1, x_2, x_3 \in \{0,1,2,3\}$

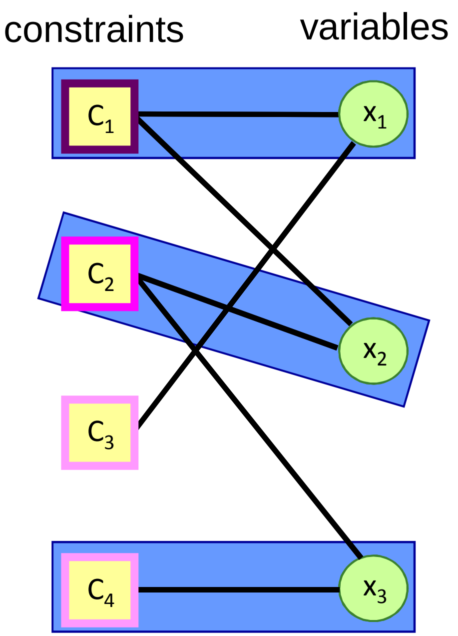
  - Constraints:

    required c_1: $x_1 = x_2$

    strong c_2: $x_2+1 = x_3$

    weak c_3: $x_1 = 0$

    weak c_4: $x_3 = 3$



3- Geometric Constraint Hierarchies

- <u>Idea:</u> Introduce preferences in GCSs in order to
  - Handle the user's sketch as a set of very weak constraints (positions, orientations, topology, …)
    → any GCS becomes over-constrained
  - Handle over-constrained GCSs by relaxing constraints automatically according to user's preferences
    → users achieve the desired solution by playing with preferences instead of debugging his constraints
- <u>Problem:</u>
  - Hoffmann et al. method does not handle preferences
  - Gangnet and Rosenberg method does not handle DOFs

3- Geometric Constraint Hierarchies

- **Proposed method:**
  - A mix of both = prioritized flow-based algorithm
  - In each iteration:
    - the introduced constraint *c* is one of the strongest ones;
    - c is distributed;
    - if it cannot be saturated, it is relaxed.

Example:

- Entities:

  Point P, Lines L_1, L_2

- Constraints:

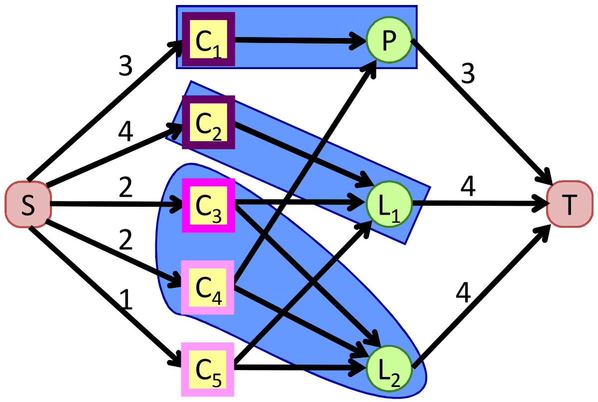
  required c1: Fixed(P)

  required c2: Fixed(L_1)

  strong c3: Parallelism(L_1,L_2)

  weak c4: Incidence(P,L_2)

  weak c5: Ortho_distance(L_1,L_2,4)



Success : c_5 relaxed!

4- Conclusions

- The proposed method:
  - achieves the LPB criterion for GCSs with preferences;
  - has the same complexity as that of Hoffmann et al.;
  - is incremental as that of Gangnet and Rosenberg;
- But:
  - it cannot include as is the “extension step” of Hoffmann et al.’s method
    → could be run in two steps: first LPB-maximal constraint set extraction, second DR-planning
  - it can return an under-constrained LPB-maximal constraint set (due to non-unary DOFs)
    → the relaxed constraints could be taken into account as an optimization criterion (satisfied as much as possible)

4- Conclusions

- How to handle the sketch then ?
  - Principle:
    - User's explicit constraints = Strong
    - Sketch = (very) Weak
      - Fixed positions
      - Fixed orientations
      - Fixed relative positions

    → Fixes remaining DOFs when the GCS is under-constrained and/or selects the "best" solution (optimization)

  => Every GCS with a sketch becomes over-constrained
  - Achieving the desired solution:
    - user indicates less/more important constraints
      → their priority is de/in-creased and the solution is updated consequently

4- Conclusions

- Future work:
  - Theoretical and practical comparison to state-of-the-art geometric constraint solvers and constraint hierarchy solvers
  - Application to other fields where constraints and entities hold multiple DOFs (Computer-aided drawing, User interfaces, …)
  - Devise user-friendly interaction schemes that render transparent the use of preferences in order to achieve the desired solution